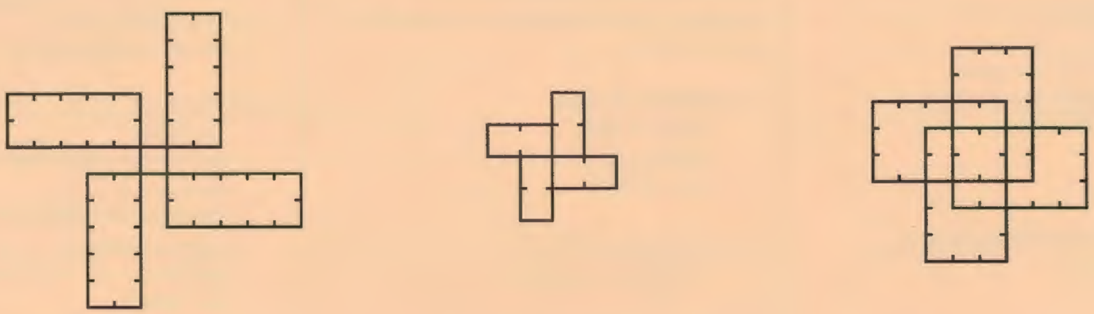
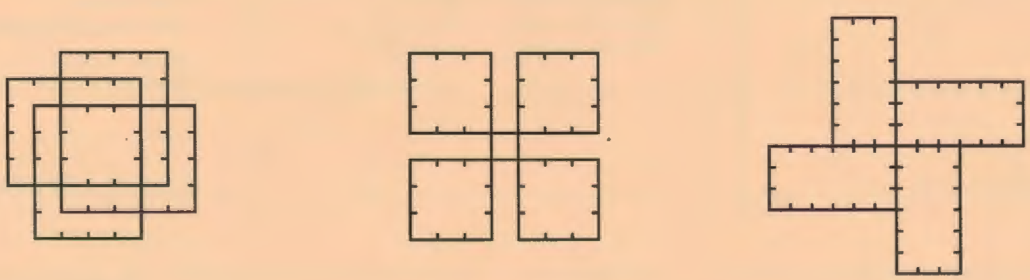
LOGO EXCHANGE

Volume 15 Number 1 Fall 1996

Journal of the ISTE Special Interest Group for Logo-Using Educators



Wumpus, Whimsy or Gloop?



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ISTE

In this issue:

Logo Exchange Volume 15 Number 1

Submission of Manuscripts

Logo Exchange is published quarterly by the International Society for Technology in Education Special Interest Group for Logo-Using Educators. *Logo Exchange* solicits articles on all aspects of Logo use in education.

Manuscripts should be sent by surface mail on a 3.5" disk (where possible). Preferred format is *Microsoft Word* for the Macintosh. ASCII files in either Macintosh or DOS format are also welcome. Submissions may be made by electronic mail as well. Where possible, graphics should also be submitted electronically. Please include electronic copy, either on disk (preferred) or by electronic mail, with any paper submissions. Paper submissions may be submitted for review if electronic copies are supplied upon acceptance.

Send surface mail to:

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Send electronic mail to:

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Deadlines

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Logo Exchange solicits articles on all topics of interest to Logo-using educators. Submission guidelines can be obtained by contacting the editor. Opinions expressed in this publication are those of the authors and do not necessarily represent or reflect the official policy of ISTE.

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Contents

From the Editor

by Dorothy M. Fitch

Welcome back!

Welcome to another year of *Logo Exchange*. This issue marks the beginning of the 15th year of this journal. I hope that you will continue to find ideas that interest you and projects to try with your students in each issue.

Patterns and motion

In this fall's issue, you can explore patterns and motion in many different ways.

Ihor Charischak challenges us to find patterns in the designs made by a simple spirolateral program. What can you and your students discover about wumpuses, whimsies, and gloops?

Nancy Flynn invites us to explore Newton's first law of motion and sine waves as we study physics using the turtle.

This leads quite naturally into ***Robert Macdonald***'s mathematical exploration of the patterns created when a billiard ball bounces around a pool table.

Jim Muller, who finds ways to use Logo to explore just about anything, gets beginners moving around the Logo screen; then we all can travel about in a geography game.

Tom Lough, who was choosen to carry the Olympic flame in Connecticut, shares this exciting event with us and makes us feel as though we were right there with him.

Don Ryoti shows how he introduces recursion to his math education students. Because many Logo users find recursion a difficult concept, it seems appropriate to present many different approaches to help us understand it.

Doug Clements and ***Julie Sarama*** show us a plan for implementing change.

Now it's your turn!

I invite and encourage you to submit your Logo projects, ideas, and teaching strategies for publication in *LX*. What are you doing with your students that excites them (and you)? What do you do that works? If you don't feel particularly talented in communicating your ideas and projects to others, I can help you. The main thing is to get started!

I hear you saying, "But I don't have the time!" Perhaps that is true. But do you have a student or a group of students who does? Would they like to share what they do with Logo in your classroom? They could describe how they came up with the idea for their project, what obstacles they encountered, how they solved them, and what they learned and discovered along the way.

They could write about a favorite Logo lesson, a robotics project, a simulation that they developed, an activity that helped them understand Logo a bit better, or how they are communicating with students in another part of the world about Logo. Be sure they include any graphics they want to share!

So, students and teachers, let me know what you are doing, how you are using Logo, and what excites you about Logo. Please share your ideas with the rest of our readers!

Here's how you get started!

Send me your article (or just a small bit of it if you want to make sure you are on the right track) by e-mail, mail, or fax (see addresses and numbers at the end of this column).

Please send your article in ASCII text format or in a Microsoft Word file for the Mac (v. 4.0) or Windows (v. 6.0). You can also send a PageMaker file for either Mac or PC.

If you have any questions, feel free to call me at 603/425-2010. I look forward to hearing from you and your students this year.

Happy Logo adventures! ▲

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Quarterly Quantum
Carry :torch Pass :flame

by Tom Lough

Carrying the Olympic flame is a moving event, in every sense of the word! On June 16th (Father's Day, no less!) I ran about one-half mile through Hartford, Connecticut, as part of the 1996 Olympic Torch Relay, holding aloft a beautifully designed and engineered torch with a flame that had originated on Mount Olympus. What a thrill!

Later, someone asked me what part of the relay made the greatest impression on me. My immediate reaction was, "Running with the lighted torch!" After all, this is the activity that took up most of the all-too-short period of time.

But, upon later reflection, I realized that there was another part of the relay that affected me much more profoundly. I suppose it did not come readily to mind because it happened so quickly. But in that brief moment, I remember experiencing a powerful emotion.

The event I am referring to is the passing of the flame. You see, the torch relay is not about carrying a torch; it is about moving the flame.

The mission of the relay was to deliver the Olympic flame to the opening ceremonies in Atlanta on July 19 in a manner that allowed as many people as possible to see it. And few came closer to the flame than those who carried the torches. Let me tell you about my encounter.

Runners for each segment of the relay were carried on ahead of the flame by a special van and deposited at designated transfer points. When I stepped out of the van with my unlighted torch, the crowd on the street began to stir. They knew that their vigil was nearly over. I could feel the excitement start to build!

After a couple of minutes, I heard the sound of sirens and applause from afar. Then the caravan of flashing lights came into view. There! In the middle of the hubbub I saw the white-uniformed figure of the runner, Ann Smith of Johnstown, Pennsylvania, holding the flaming torch aloft!

At about that same instant, a technician on a motorcycle roared up and turned on the gas supply for my torch. I could hear the energetic hissing as the gas escaped the burner in search of a flame.

In a few moments, Ann reached my position and paused a couple of meters away. Slowly and dramatically, she raised her flaming firebrand up into the air and then extended it forward. In response, I raised my own torch and moved it toward hers.

Whoooooosh!

The flame leaped across the gap between us, completing the arch we formed and engulfing the top of my torch. What a moment! It was simply beautiful!

The run was a blur, even though I tried to go slowly and show the flame to everyone along the way. All too soon, I saw the next runner, Bob Mayo of Granby, Connecticut, up ahead, waiting with his unlit torch.

As the flame leaped eagerly from my torch to his, I could feel a surge of the Olympic spirit passing through me to him as well. It was these moments of passage that made a deep impression on me.

I later realized that I had experienced a similar feeling several times before: when a Logo student I was helping suddenly smiled and surged on ahead, and when a Logo teacher I was assisting began to get excited about what this computer language could empower her students to do.

All of us are carrying the torch for Logo as we go about our day-to-day teaching. But it is also important to realize that we must pass the flame on as well. The embodied spirit of Logo—the excitement and thrill of this special type of learning—must be an integral part of our teaching activity.

Even though we each pass the Logo flame to others in different ways, let's become more aware of these special moments. It is in these moments that lives are changed in ways we cannot imagine.

And isn't that what teaching is all about? ▲

FD 100!

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Experiment: Online code!

This year, most of the programs printed in *LX* will also be available electronically. This should save you lots of time (no more entering long programs) and help prevent bugs (the kind that creeps in through typing errors).

The programs will be exactly as listed in *LX*. However, if any readers would like to convert the programs to other versions of Logo and send them back to me electroncially, I can then make them available to others on request.

In this issue, the following Logo programs are available electronically:

- **SPIROS**
  from "Wumpus, Whimsy, or Gloop?"
- **POOLBALL**
  from "The Mystery of the Moving Billiard Ball"
- **PHYSICS**
  from "Logo in Motion"
- **TEXAS**
  from "Exploring Turtle Geography"

To receive one or more of these listings electronically, send a request to me at 71760.366@compuserve.com with the name of the program(s) you would like to receive (use the bold name listed above). Also indicate which version of Logo you are using. If that specific version is available, you will receive it; otherwise you will receive the version listed in *LX*.

I hope you will find this new service useful. As always, I welcome your comments and suggestions.

Dorothy Fitch, *LX* editor
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Logosium '96

reported by Theodore Norton

Logosium lives! On June 14, 1996, Marian Rosen and Michael Tempel opened the third annual Logosium, sponsored by the ISTE SIGLogo and The Logo Foundation, in an atmosphere of cordiality created by members and friends of the St. Paul Logo Project. Logosium was hosted by the St. Anthony Park Elementary School in St. Paul, Minnesota, an inviting site that is well endowed with computer resources.

The 85 participants were drawn from the local area, from the rest of the country, and from around the world (Mexico, Canada, Brazil, Australia, Saudi Arabia, and Southeast Asia). There were four sessions with five to six topics each, including open labs. The topics, together with their student and adult facilitators, included:

- *We're Right, They're Wrong* with Gary Stager
- *Logo-Based Multimedia Projects* with Sharnee Chait and Dorothy Fitch
- *Logo on the World Wide Web* with Michael Tempel
- *Expert Mathematician—Power Math Logo* with Jim Baker and Dale Hulme
- *Getting Emotional With Lego & Logo* with Keith Braafladt and Natalie Rusk
- *Game Design & Construction—Teachers* with Hope Chafiian and Michael Tempel
- *Game Design & Construction—Kids* with Jake Bruer and Michael Petersen
- *Visual Modeling With Logo* with Darrell Mohrhauser
- *Maya Quest: Logo With CDs & Videodiscs* with Kathy Ames and Greg Anderson
- *Logo & ESL* with Charles Lee
- *Logo & Robotics* with Dorothy Fitch and Paul Krocheski
- *Logo Supports Formal Geometry and...* with Kassidy Newscom, Barbara Sylvester, Breana Sylvester, and Danielle Wells
- *...Not So Formal Multicultural Geometry* with Nancy Flynn
- *MicroWorlds Projects About Inventions* with Eleanore Bednarsh and Joan Nelson
- *The Nitty Gritty of Importing Graphics* with Ron Beck
- *The Programmable Brick* with Michael Tempel and David Tempel
- *Logo in an Elementary School* with Charlotte Coan and Judy Steiner
- *Logo Animation Techniques* with Bill Spezeski
- *Logo and Laptops—When is Enough?* with Diane Jackson, Paul Krocheski, and Marian Rosen
- *Myst & MicroWorlds* with Keith Braafladt and Natalie Rusk

As this list suggests, a wide range of interests, from telecommunications to "living" computerized environments, was represented, as were different Logo-based systems: MicroWorlds was put through its parallel processing paces, while Terrapin and PC Logo-driven robots whizzed and whirled across laboratory floors. Children made exciting and articulate presentations, as their mentors pondered Logo's future.

A high point of the day was the keynote address delivered by Geraldine Kozberg, founder of the St. Paul Logo Project, on "Whatever Happened to the Revolution?" Ms. Kozberg recalled the heyday of "classical" Logo in 1984–1986: Brian Harvey's posing of her title question in 1986 and her reply, "We are a culture in transition, not a revolution." Ten years later, according to Ms. Kozberg, the nonrevolutionary character of the Logo movement has been confirmed. However, the problems of the present are posed not by technology but by the denial of social justice. Once again we must build the social and behavioral prerequisites for effective learning for all children, even as its context is lost to technocentric commerce. Accordingly, the emphasis of the next 15 years must be placed on equity and the dynamics of learning environments.

Next year, Logosium is again planned to take place in conjunction with NECC, this time in Seattle, Washington. Watch for information about Logosium '97 in future issues of *LX*. ▲

The Beginner's Corner: Exploring the Logo Screen

by Jim Muller

As we begin another year, it might be useful to review some of the ways we can help Logo novices get excited about Logo. Practice in the basic commands is essential. But how can you keep this exciting? Hopefully you will find a new idea or two in this article on "getting started."

Getting started

During the spring and summer of 1981, my son and his friends would meet at our house on Saturday mornings to explore the wonders of TI Logo, the first commercial Logo package. Two of the boys were "forced" into dragging their young siblings along. Although this was a chore for the boys, it provided the spark that ignited my interest in kids, computers, and Logo.

While the older boys were developing games, videos, and other more exotic procedures, I started the young children off by letting them explore what the turtle could do. Some weeks later I got curious. I asked the younger children to put their pencil in the middle of a piece of blank paper and go forward 100 turtle steps, right 153 turtle turns, and forward 100 again. On other pieces of paper, I asked them to turn other random angles: 221, 47, 305, for example.

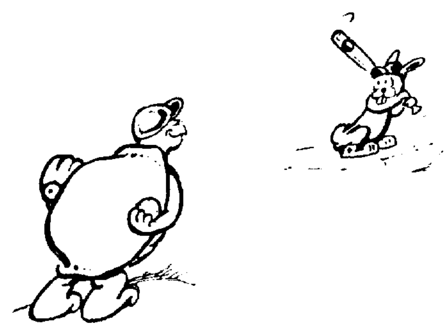
Later, I asked three design engineers to sketch a similar set of random angles. Both they and I were amazed to see that all four of those young children, ages 6, 8, and 10, were able to more accurately visualize random angles than were trained designers. The designers acknowledged they had little to no practice in visualizing random angles like that. They relied on readily available tools.

How did this happen?

To get the young children started in Logo while the older boys and I focused on the "more important stuff," I used an erasable marker to sketch some flowers and rocks on the computer screen. After giving the children some practice with the basic turtle commands, we made up a game.

Who could help the turtle find the flowers without bumping into a rock in the least number of moves? We could just as easily have made this into a game of golf. Who could find the first hole in the fewest stokes?

Finding the flowers led to a more challenging game. When the youngsters had a bit more experience with angles and distances, we started the Turtle Baseball League. This was a very popular group activity when we later began to visit Logo classes and computer clubs.

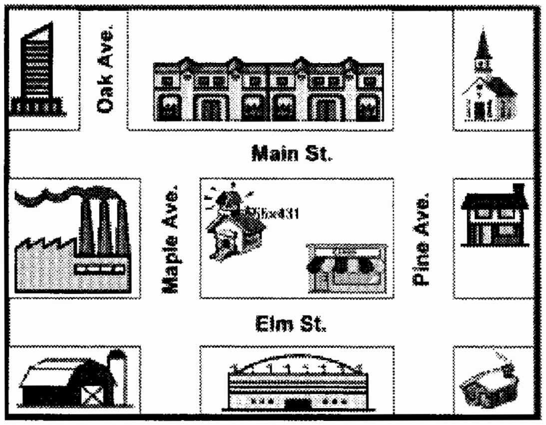


We drew four bases on the screen. Each player had three chances—three strikes—to get to first base or they were out. Then the next player on their team was "up."

Players who got to first base in less than three strikes could use their remaining strikes to continue around the bases. At first, the bases were equidistant from one another. When this game was mastered, random distances and angles were added.

Exploring "Turtle Town"

Taping transparencies to the computer screen was another way of taking the drudgery out of developing visualization skills. Exploring Turtle Town was among the more popular exercises, largely because it offered so many options for different kinds of learning fun.



The first tasks were simple exercises with directions and distances.

1. Start from your home in the lower right-hand corner and find the farm.
2. Now find the factory.
3. Now it's time to go to school.

Then came the challenges. Who could complete a trip in the least number of moves? For example:

1. Go to your friend's house.
2. Take lunch to your friend's father who works at the factory.
3. Go see your cousin who lives in the apartments.
4. Go get some candy at the candy store.
5. Go back home.

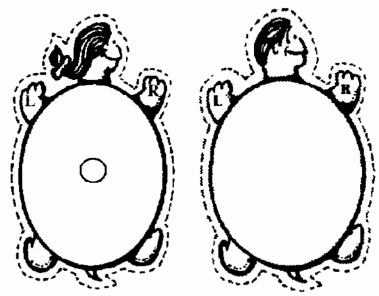
Turtle geography

One nice thing about transparencies is that you can make paper copies. Children have a chance to explore Logo off the computer using a pencil or "turtle" marker. This is also a way of introducing the subject of turtle geography through the maps of Turtle Town. Forward, back, left, and right now take on new meanings: north, south, east, and west. For example:

1. Go north on Pine Avenue to Elm Street.
2. Go west on Elm Street to Maple Avenue.
3. Go north on Maple Avenue to Main Street.
4. What direction must you turn to find the skyscraper at the corner of Main and Oak Streets?

One of the transitions that young children invariably have trouble with is visualizing turtle commands when the turtle is heading south, toward the bottom of the screen.

Making turtle pencils has been a big help. Paste turtles like the ones shown below on a piece of cardboard.

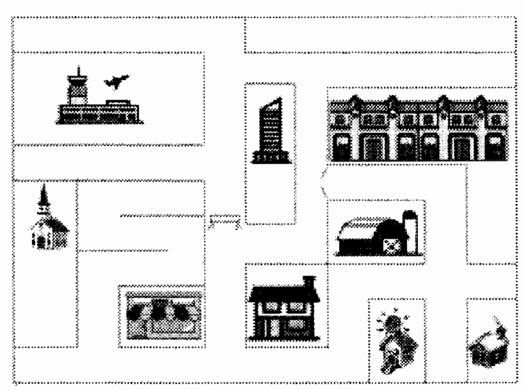


Then push the turtle onto a pencil, leaving enough room so that the child's hand can fit underneath. The trick is to have the pencil mimic the turtle.

Go forward 100. (The turtle moves toward the top of the paper.)
Right 90. (The turtle faces the right edge of the paper.)
Forward 100 (The turtle moves toward the right edge of the paper.)
Right 90 (The turtle faces the bottom of the paper.)

The L and R on the turtle's front feet help keep directions straight.

You can take the concept of Turtle Town maps just as far as you want. The second map shows you a few ideas.



- Add some variables to the map, such as temporary construction barriers.
- Turn the map into a maze.

- Have the children create their own Turtle Town game.

Game design

Divide the class into several teams. Each team must then design its own Turtle Town game. When completed, teams play the games developed by each other and vote on which is the best game. This activity can also be shared with other classes in the same or another school.

Game design can be very useful. Not only does it force teams to think through a series of Logo commands, but it also forces them to think logically and strategically. The same goes for players. They must be equally as creative to win. For example:

- One team added the feature that any player who drove off the road had to go to jail and miss three turns.
- Members of another team developed a game-winning strategy: They turned the turtle into a helicopter so they could get across town directly, rather than move through the streets and make all those turns. (All they did was lift the pen up.)

Working with maps and developing games lead to all sorts of wonderful possibilities. Hopefully the ideas offered in this article will lead to many more of your own. If you are working with older children, why not take a look at the "Logo Geography" article in this issue. It offers methods for bringing the subjects of maps, map making, geography, and turtle geometry together. ▲

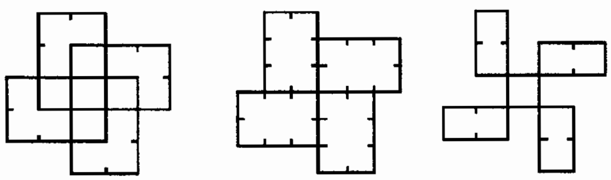
Jim Muller has had a lifelong interest in translating various technologies into understandable and persuasive programs. In 1981, Muller and his son organized the first Logo users group, the Young Peoples' Logo Association, which eventually grew into a worldwide 6,000 member organization. In 1985, the YPLA merged with CompuServe, where it became The Logo Forum. Today, Muller is a computer training and marketing consultant in the Dallas/ Fort Worth metroplex. Look for Jim's ideas and activities in the newly updated *The Turtle's Discovery Book*, an expanded two-volume set to be published by Harvard Associates, Inc. later this year. You can reach Jim by e-mail at 76703.3005@compuserve.com or on CompuServe at 76703,3005.

Wumpus, Whimsy, or Gloop?

by Ihor Charischak, President of CLIME (Council for Logo & Technology in Math Education)

Spiros (from *CLIME Microworld*, Vol. 1) continues to be one of my favorite activities in teacher workshops and with students. I've done such a session many times and I find that I learn something new every time.

Spiros draws spirolaterals that can be categorized as Wumpuses, Whimsies, or Gloops.



Wumpus Whimsy Gloop

Experimenting

The main procedure **S** takes three integers that are less than 20 as input. Go ahead—try various inputs to **S**. Look at the designs and collect data. Initially there appears to be no correlation between the numbers and the design. But after some looking and thinking, patterns emerge. See the CopyMe! page on page 10.

Can you figure out what makes a Wumpus a Wumpus, a Whimsy a Whimsy, and a Gloop a Gloop? That is the first step.

After my workshop participants have used the **S** procedure to generate various designs, I ask, "Is there any relationship between the designs (Whimsy, Wumpus, and Gloop) and the numbers? Can you predict what the shape will be from the numbers?"

Conjecturing

Teachers come up with all sorts of conjectures, some of which are fruitful and others less so. For example, one group of three teachers noticed that **S 1 1 1**, **S 2 2 2**, **S 3 3 3**, and so on formed squares that were always Wumpuses. Thus, their initial conjecture was: If all the numbers are the same, you get a Wumpus.

This was correct, but it didn't seem to help them in situations when the numbers are not all the same. So they decided to see what happens when they keep two numbers the same and vary the third (Figure 1). No light bulbs went off immediately. They were all surprised that **S 2 2 5** was a Gloop. I walked away from the group and then returned a few minutes later, noticing that one of them was exploring Gloops. She eventually came up with a pattern that she generalized to all three categories.

Arriving at a solution

She noticed that **S 2 2 5** produced a Gloop that had a 1 x 1 square in the middle. She then looked at the other Gloops (see the Copy Me!

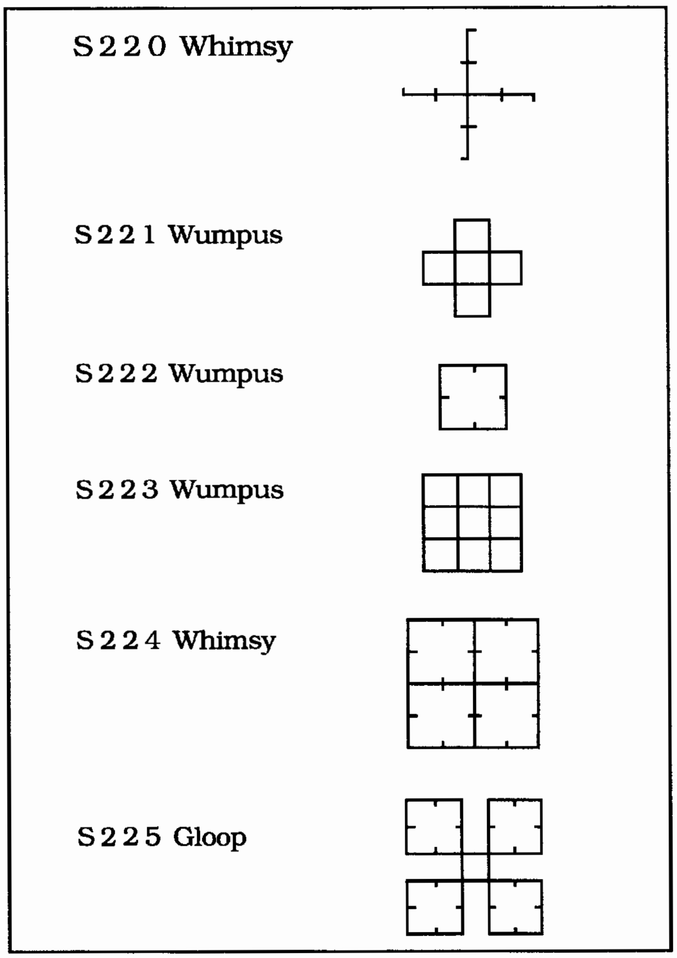
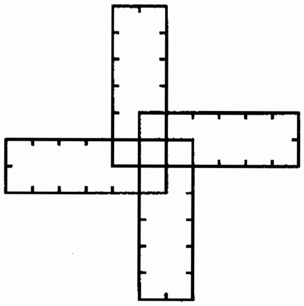
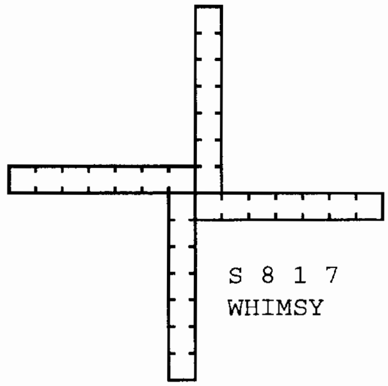


Figure 1

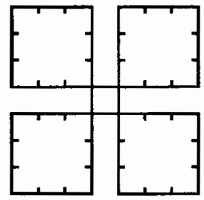
Spiros Data



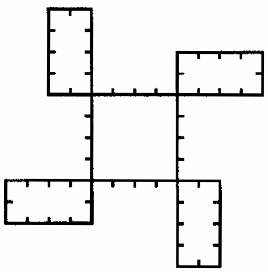
S 2 7 6
WUMPUS



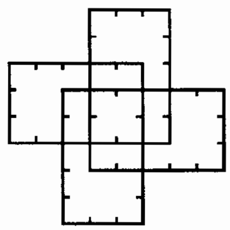
S 8 1 7
WHIMSY



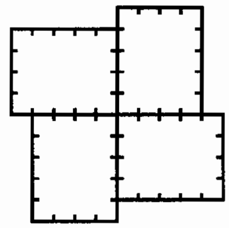
S 3 3 7
GLOOP



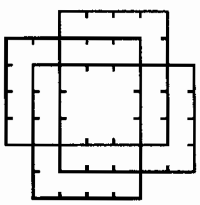
S 10 4 2
GLOOP



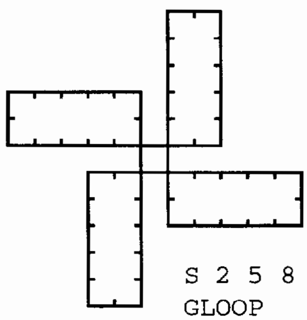
S 3 5 6
WUMPUS



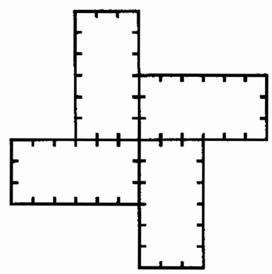
S 4 5 9
WHIMSY



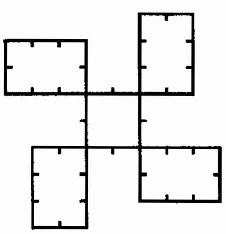
S 6 4 5
WUMPUS



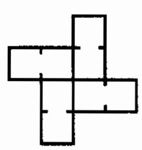
S 2 5 8
GLOOP



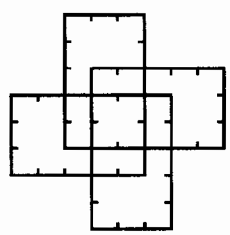
S 3 9 6
WHIMSY



S 3 7 2
GLOOP



S 1 2 3
WHIMSY



S 5 3 6
WUMPUS

page) and recorded the information in a table. It wasn't long before she discovered a pattern for Gloops that she was able to extend to Whimsies and Wumpuses.

See if you can discover how she did it! Think about it before looking at her conjectures on page 34.

The program

This program was written using Terrapin's Logo PLUS for the Macintosh. It can be easily translated to other versions of Logo.

After entering all the procedures, type **startup** to begin.

```
to startup
cleartext
print []
print [SPIROS]
print []
print [Type S followed by three single digit
  numbers. These numbers represent the
  distances of three forward motions.]
print []
print [A right turn will be made after each
  forward motion. This pattern will repeat
  until the path closes.]
print []
end

to s :first :second :third
error.trap scale
spirolateral :first :second :third
end

to spirolateral :first :second :third
draw hideturtle
penup setx -60 pendown
repeat 4 [fwd :first right 90 fwd :second
  right 90 fwd :third right 90]
report.type
end

to fwd :distance
repeat :distance [forward :scale tick.mark]
end

to tick.mark
right 90 forward 2 back 2 left 90
end

to scale
if (or :first > 16 :second > 16  :third >
  16) [make "scale 4 stop]
if (or :first > 8 :second > 8    :third > 8)
  [make "scale 8 stop]
if (or :first > 4 :second > 4    :third > 4)
  [make "scale 10 stop]
make "scale 12
end

to error.trap
if :first > 19 [warning :first toplevel]
if :second > 19 [warning :second toplevel]
if :third > 19 [warning :third toplevel]
end

to warning :number
print sentence :number [is too large an
  input, please reenter with all inputs less
  than 20.]
end

to report.type
sort.inputs
if :s + :m > :l [print [This is a WUMPUS.]]
if :s + :m = :l [print [This is a WHIMSY.]]
if :s + :m < :l [print [This is a GLOOP.]]
end

to sort.inputs
if :first > :second [make "l :first make
  "others [second third]] [make "l :second
  make "others [first third]]
if :third > :l [make "l :third make "others
  [first second]]
if thing first :others > thing last :others
  [make "m thing first :others make "s thing
  last :others] [make "m thing last :others
  make "s thing first :others]
end
```

In addition to founding CLIME in 1987 and supporting it ever since, ***Ihor Charischak*** is program manager for the Center for Improved Engineering and Science Education (CIESE) at Stevens Institute of Technology. His latest project is managing a 3-year NSF-funded mentorship project that involves 40 math teachers (Grades 7–10) from 16 school districts in New Jersey who have been learning to use technology in their classrooms and to share their knowledge with their colleagues. For more information about CLIME you can write to Ihor at 10 Bogert Avenue, White Plains, NY 10606 or send e-mail to icharisc@stevens-tech.edu.

The Mystery of the Moving Billiard Ball

by Robert Macdonald

The idea of creating a statistical and data-collecting microworld dealing with the path of a billiard ball was the result of my coming across a group of activities in a book by Harold R. Jacobs (1970).

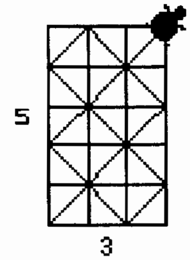
The ordinary billiard table is about half as wide as it is long, some 5 by 10 feet. Jacobs suggests stylizing the game to produce a delightful series of puzzles, for which he merely intimates solutions.

Students should have some prerequisite skills before undertaking this microworld. First, they should have sufficient skills with Logo software, in this case, LogoWriter. They should be able to move in and out of programs, give commands with or without inputs, add labels, and print material with relative ease. In addition, students should have had enough experience in collecting data and utilizing that data to make sensible predictions and arrive at well thought-out conclusions. It is essential that students have some skill in reducing ratios to their lowest terms. I have explored this microworld with fourth-grade students.

The setup

Suppose we are to set up a billiard table in the form of a grid made up of squares. The only ball is struck from one corner so that it always moves at a 45-degree angle to the sides of the table. As it strikes a wall, the ball is always deflected from its path at a 90-degree angle until it exits on arrival at one of the corners of the table.

For example, let's take a table with a length of five units and a width of three units. Suppose we enter at a 45-degree angle at the lower left corner. The ball will move forward on a diagonal through three squares before hitting the wall and deflecting to the left at a 45-degree angle to the wall (but at a right angle to its own path). It then moves forward through two more squares until it is again deflected left at a 45-degree angle to the wall. Following the path of the billiard ball in Example 1, we note that it strikes the wall four more times prior to exiting at the upper right corner.



Example 1

The program

To provide a computer experience in constructing a billiard table and playing on it, a program is essential. The following procedures, written in LogoWriter for the Macintosh, have proven effective.

```
to grid :length :rows :columns
clearpage
place
repeat :rows [row :length :columns go.back]
seth 0
st
hypotenuse :length
move
end

to clearpage
if not front? [flip]
rg ht ct cc
end

to place
pu setpos [-192 77] pd
end

to row :length :columns
repeat :columns [square :length forward
  :length]
end

to square :length
seth 90
repeat 4 [forward :length right 90]
end
```

```
to go.back
seth 270
forward :length * :columns
seth 180
forward :length
end
to hypotenuse :length
make "hypotenuse sqrt (2 * (:length *
  :length))
end

to move
move.ball readchar
end

to move.ball :letter
if equal? :letter "c [right 45]
if equal? :letter "f [forward :hypotenuse]
if equal? :letter "b [back :hypotenuse]
if equal? :letter "r [right 90]
if equal? :letter "l [left 90]
if equal? :letter "u [pu]
if equal? :letter "d [pd]
if equal? :letter "e [pe]
if equal? :letter "s [stop]
move.ball readchar
end
```

Analyzing the program

The main procedure **grid** draws squares representing a billiard table upon the screen. **Grid** requires three inputs: (a) the length of each square in turtle steps, (b) the number of rows (length) desired in the table, and (c) the number of columns (width) in the table.

For our purposes, 15 turtle steps provides a good size for the side of each square. For practical purposes, a 12 x 12 grid would most likely be the largest grid desired by most users, although students very likely will want to experiment with larger sizes. A setup subprocedure places the turtle before the grid is drawn.

Because a billiard ball (the turtle) will be traveling on the diagonal (hypotenuse) of a square, we need a procedure to determine the length of that hypotenuse. In the procedure **grid**, the hypotenuse takes the input of the length of the square.

Now we must move the turtle (billiard ball) around the grid (billiard table). Keys can be programmed to move the ball on command. This is accomplished by using the procedure **move**.

The ball will always enter from the lower left corner. The key C sets the turtle in the lower left corner of the grid at a 45-degree heading, which permits it to move on the hypotenuse produced by the diagonal cutting through each square. The 45-degree heading is produced by the command **seth 45**. The key F moves the turtle forward the distance of one hypotenuse. Key B moves the turtle backward the distance of one hypotenuse. Key R produces a right rotation of 90 degrees. Key L produces a left rotation of 90 degrees.

Because errors may have to be corrected, four additional keys are helpful: Key U creates the command Pen Up; Key D produces Pen Down; Key E produces Pen Erase; and Key S, Stop, provides an exit from the **move** procedure. If you forget to provide that key, after you have drawn your grid and traced the action of the ball, you will not be able to get out of the program and move forward to do the necessary labeling.

After creating a table, it is possible to label it. This labelling might include dimensions, exits, and the number of bounces the ball takes off a side (the wall of the billiard table).

Providing an instructional sheet for students might initially prove helpful. Be sure to give a sample **grid** instruction:

```
grid 15 10 4
```

15 = the side of each square in turtle steps
10 = the number of rows
4 = the number of columns

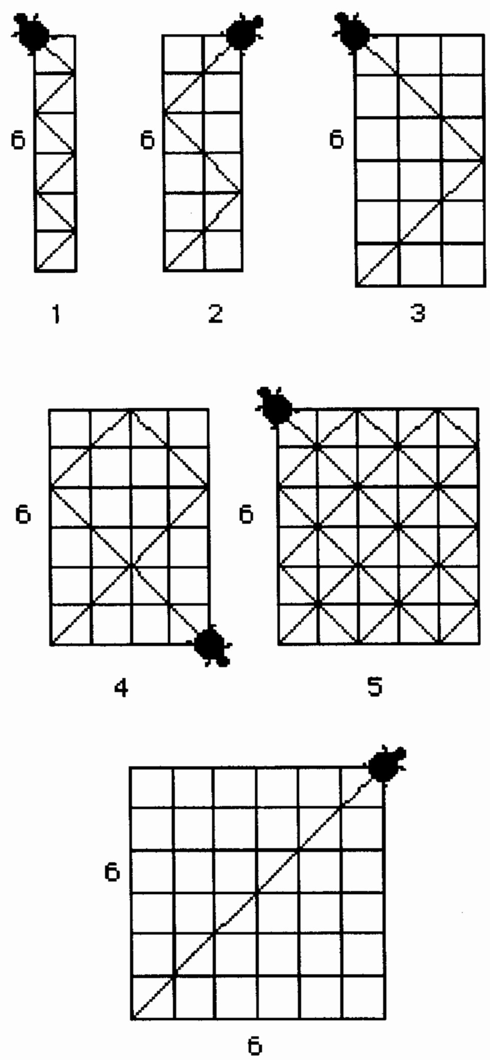
In addition, provide an interpretation for the keys available in the **move** procedure.

C = start (changes the turtle's heading from 0 to 45, equivalent to **right 45**)
F = Forward :hypotenuse
B = Back :hypotenuse
R = Right 90 **D** = Pen down
L = Left 90 **U** = Pen up
S = Stop **E** = Pen erase

You may need to review how to label your grid. In LogoWriter for the Macintosh, merely click the mouse on the work area and use the cursor to position your work. On an Apple IIe or IIGS you must know how to use Label mode and Turtle Move mode.

Preliminary observations

If we were to draw grids indiscriminately, the paths of the ball would appear to be markedly unpredictable. A slight change in dimension may cause a wildly unexpected path. It is apparent that the shape of the table foreshadows the path of the ball in ways that are still not clear. But what determines the shape of the table? The length and the width. If we change these dimensions, we introduce the concept of variables. Thus the path that the ball will take is predisposed by the shape of the table, which is fixed by its dimensions.

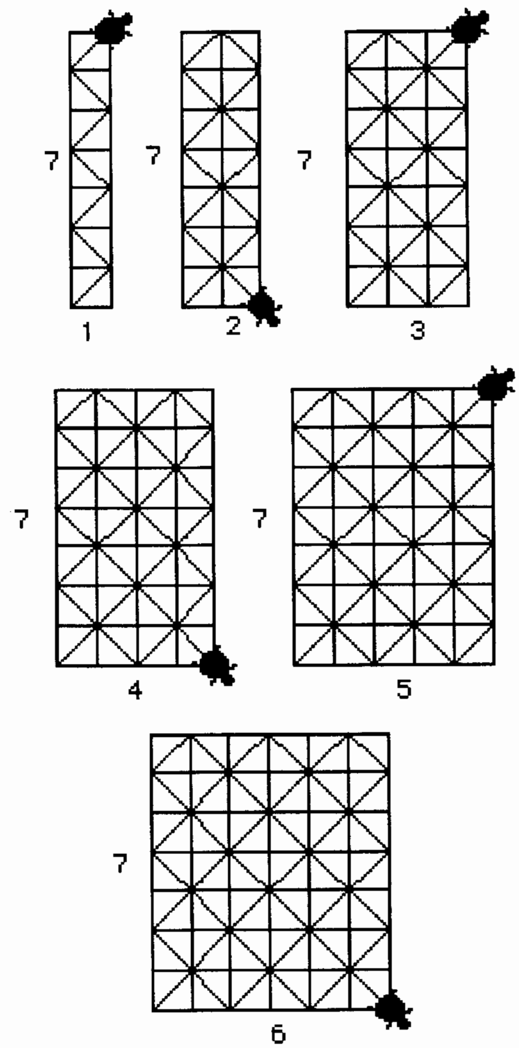


Example 2

It would make our investigation far simpler if we had just one variable. Thus, let us keep the length constant, but vary the width.

If we were to draw a set of six billiard tables with a length of six units, but vary the widths by 1, 2, 3, 4, 5, and 6 units, the results prove interesting. What dimensions (length by width) produce the simplest path? The most complicated path? What can we determine about the number of bounces off the walls? (See Example 2.)

If we were then to draw a set of seven tables with a length of seven units but vary the widths by 1, 2, 3, 4, 5, 6, and 7 units, the results differ markedly from the preceding set. In many tables the turtle traverses every square. Is it possible to deduce which tables those will be? (See Example 3.)



Example 3

These billiard grids provoke all sorts of questions that relate to grids of varying dimensions. Will the ball (turtle) always exit at a corner, or will it rebound from the walls eternally? Will the ball ever return to the original point of origin? If the ball does exit at a corner, and you know the length and width of the grid, will it be possible to predict the exit point and the number of bounces to ensure that exit? Will it be possible to predict the total number of right and left bounces that the ball will make during its journey?

Methodology

Hands-on activities with graph paper and computer-related exercises producing billiard tables may provide ample statistical evidence with regard to exits, number and types of bounces, and patterns of journeys. Drawing conclusions through observation and noting regularities and irregularities in patterning create a wonderful microworld for inductive reasoning.

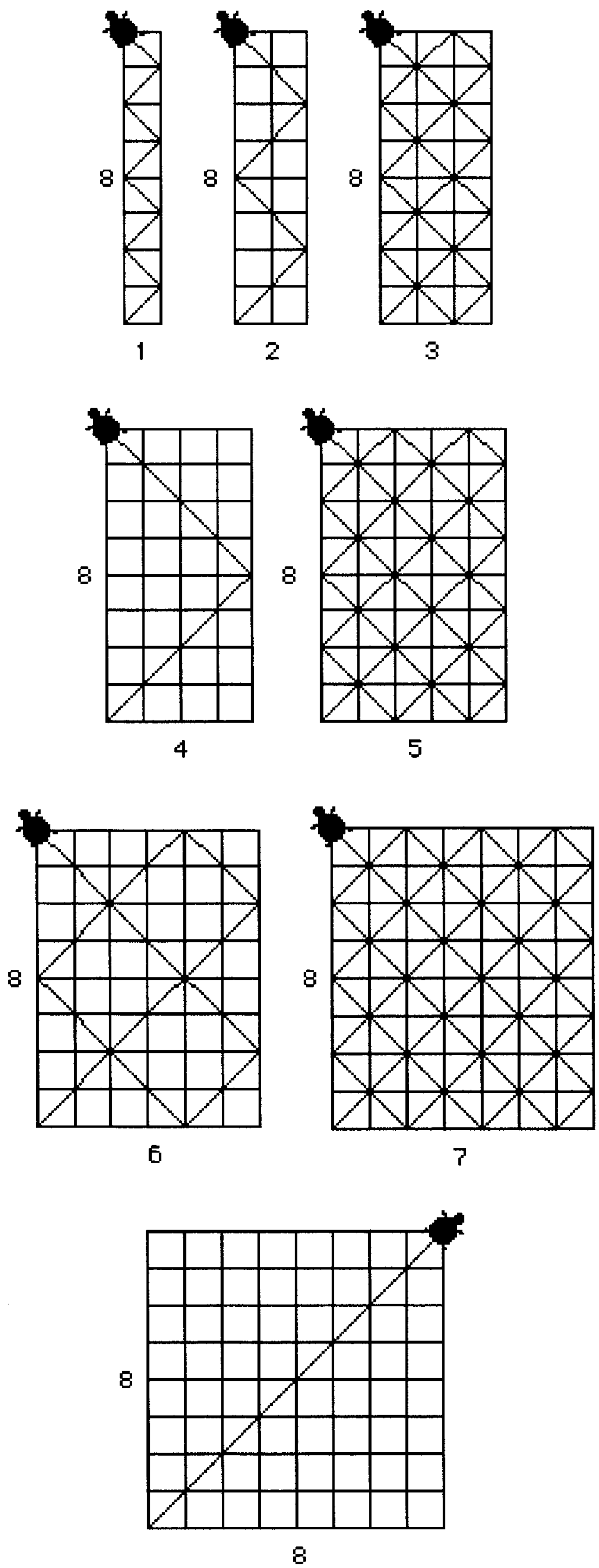
Although inductive reasoning is important in clarifying mathematical ideas, there is a basic weakness. If we draw our evidence from a limited base, we run the risk of drawing conclusions from limited data. Thus, we must be certain that we have a wide enough spread to reduce the risk of predicting possibilities from a faulty set of data.

For example, we could deduce that if we were to draw a set of tables with a length of 8 (8 x 1, 8 x 2, 8 x 3, etc.), the ball would always exit in the upper left corner. This is true up through 8 x 7. However, 8 x 8 produces a marked change, hence the need for a wider spread of data. (See Example 4.)

The concept of the hypotenuse

The turtle's path on our billiard table is along the diagonal of each square. This path, the hypotenuse, is the side of a right triangle that is opposite the right angle.

Students should be given some explanation about the hypotenuse and how it relates to the length of the side of the square. The most direct approach might be to draw a large square with its diagonal. Simple measurement of a side with a comparison to the measurement of the diago-



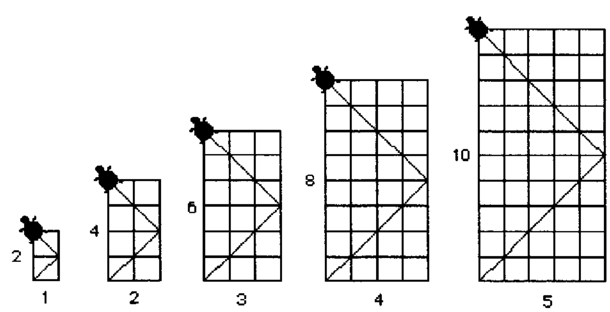
Example 4

nal should prove the need for a formula to compute the distance of the hypotenuse. More able students may examine the computer procedure to discover the formula. Most students will be satisfied with simply measuring distances.

The concept of the ratio

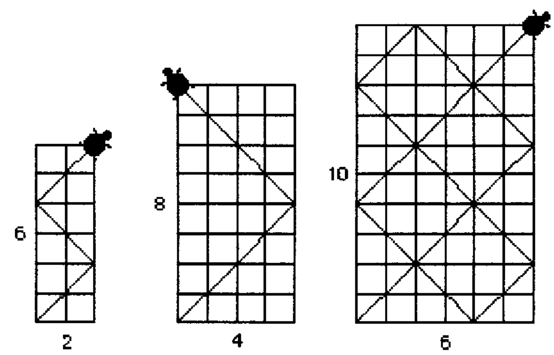
A ratio is a fraction. It is a way of making comparisons between two numbers. A reduction of these numbers by division, producing a simpler ratio, may open up interesting relationships. To discover the simplest ratio between two numbers, divide both by their largest common factor. For example: 6/3 = 2/1.

In this microworld, simplifying ratios is easily demonstrated through tables. Students quickly discover that the path of a ball depends upon the shape of the table. If the shapes of tables are the same, paths should be identical. Discover this through graphing. In Example 5, the ratios are identical. 2/1 = 4/2 = 6/3 = 8/4 = 10/5; thus, all paths are identical.



Example 5

Are the paths in Example 6 similar? No. Why not? The simplest ratios are not equal. 6/2 = 3/1, 8/4 = 2/1, 10/6 = 5/3. The ratio 3/1 is not equal to either 2/1 or 5/3.



Example 6

The study of the path of a billiard ball on a table deals with mathematics, but is not entirely devoted to calculations. Simple addition and the reduction of ratios are just basic requirements. Yet with these skills, students can do surprising things.

Student participation

Each student should have ample opportunity to gather and record statistical information on a full gamut of billiard tables. Generally, a series of lengths from 1 through 10 with widths from 1 through 12 to 15 should represent a broad enough sampling to provide sufficient evidence for making predictions and arriving at sensible conclusions.

Students can record data for each trial in the following columns:

- dimensions of the table
- sum of the dimensions
- number of bounces
- exit corner (UL, UR, LR)

In addition, each child should keep a personal account of graphing and predictions. Booklets of computer printouts of billiard tables and large wall charts of each series should be posted for class use, along with laws that are postulated.

The bounce

The class initially viewed bounces in two ways. Some students preferred to count only a hit off a wall as a bounce. Others considered an entry, a hit off a wall, and an exit as valid bounces. We settled on the latter.

Definition of a bounce:

1. Count the entry (lower left corner).
2. Count the exit.
3. Count each hit off a side.

Law for the number of bounces

After defining the bounce, students can investigate how many bounces varying tables can produce. It may be easier to have the investigation begin with a constant length of one and vary the width, for example, 1 x 1,1 x 2, 1 x 3, 1 x 4, and so on.

Children will soon discover that adding dimensions will total the number of bounces for each table. However, if we introduce the series of threes (3 x 1, 3 x 2, 3 x 3, 3 x 4, etc.), irregularities appear. For example, a table of 3 x 3 does not have six bounces, but rather two. A 3 x 6 table does not have nine bounces, but three. If children have had sufficient experience in reducing ratios, they soon are aware that a reduction in ratios solves the problem. Reinforcement by continuing to project solutions to odd-numbered lengths should be encouraged.

When the discovery of reducing the ratios of dimensions has been assimilated, the investigation of even-numbered lengths is much easier to handle.

Law of the number of bounces:

1. Reduce the dimensions to the lowest ratio.
2. Add the dimensions together.

By this time students should be able to create projection sheets and determine with some accuracy the number of bounces for a complete series of billiard tables. They should be encouraged to check their predictions by graphing solutions. You may wish to put each series on a different color of paper to make organizing data much easier.

Laws of exits

Students will have produced collectively a body of statistical data and visual information to permit the discovery of the laws of exits.

Again, it may be easier to start with the 1x series. All of these tables exit either to the upper right (UR) or lower right (LR). Projection sheets are now essential. Have the students compile them and prove them by graphing.

In the 2x series we discover exits to the upper left (UL), upper right (UR), and lower right (LR). None exit where the turtle enters.

In the 3x series we find that all exits are to the UR and LR. By this time some students may deduce that the same will be true of all odd length series (5x, 7x, 9x, etc.). It is the even length series that produces variety. Why? Experiments will prove that exits for 4x, 6x, and 10x are to the UL, UR, LR, but never to the lower left (LL). The 8x series will exit only to the UL and UR. There no exits at the point of entry nor at the LR.

It will be apparent to many children that the reduction of dimensions to lowest ratios and whether these relationships may be considered odd or even numbers play an important part in bringing order out of seeming chaos.

Through observation a class may determine these laws of exiting:

Law 1: If table length is an odd number and the width is even, the turtle exits to the Lower Right (LR).

Law 2: If table length and width are odd numbers, the turtle exits to the Upper Right (UR).

Law 3: If table length is even and the width is odd, the turtle exits to the Upper Left (UL).

Law 4: If both table length and width are even, reduce the ratio of the dimensions to the lowest terms and follow Law 1, 2, or 3.

This may be shown concisely as follows:

Laws of Exits

	Length	Width	Exit
Law 1	odd	odd	UR
Law 2	odd	even	LR
Law 3	even	odd	UL
Law 4	even	even	

Reduce Ratio to Lowest Terms

	Length	Width	Exit
Law 1	odd	odd	UR
Law 2	odd	even	LR
Law 3	even	odd	UL

Patterning

Eventually students should be drawn to examining patterns. In their study of ratios they have discovered that similar patterns may be masked by unreduced ratios. In addition, some students may be confused by the rotation of a pattern. This posed little difficulty for many of my classes because the students had had much experience examining tangrams and pentominoes.

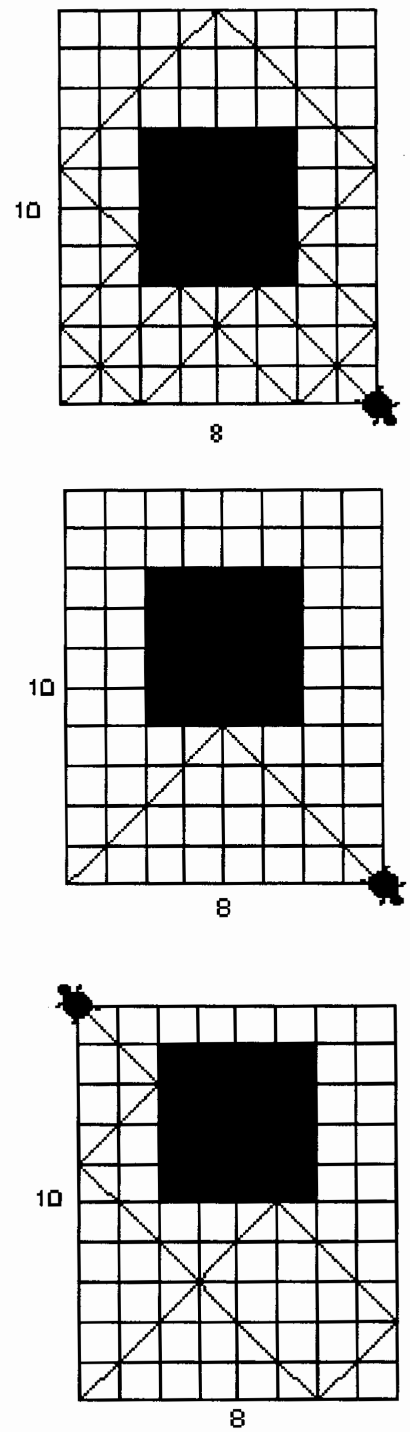
Some classes developed pattern sheets to record similar patterns. The students discovered that the following ratios were pertinent to the study: 1/1, 1/2, (2/1), 2/3, (3/2), 3/4, (4/3) 3/5, (5/3). Students were quick to recognize patterns that were simply extensions of dimensions.

Further explorations

What if an entry point other than the lower left were introduced? What if exit points other than corners were permitted? What if we blocked off part of the playing area of a billiard table? Could we predict the number of right and/or left bounces if we knew the dimensions of a table?

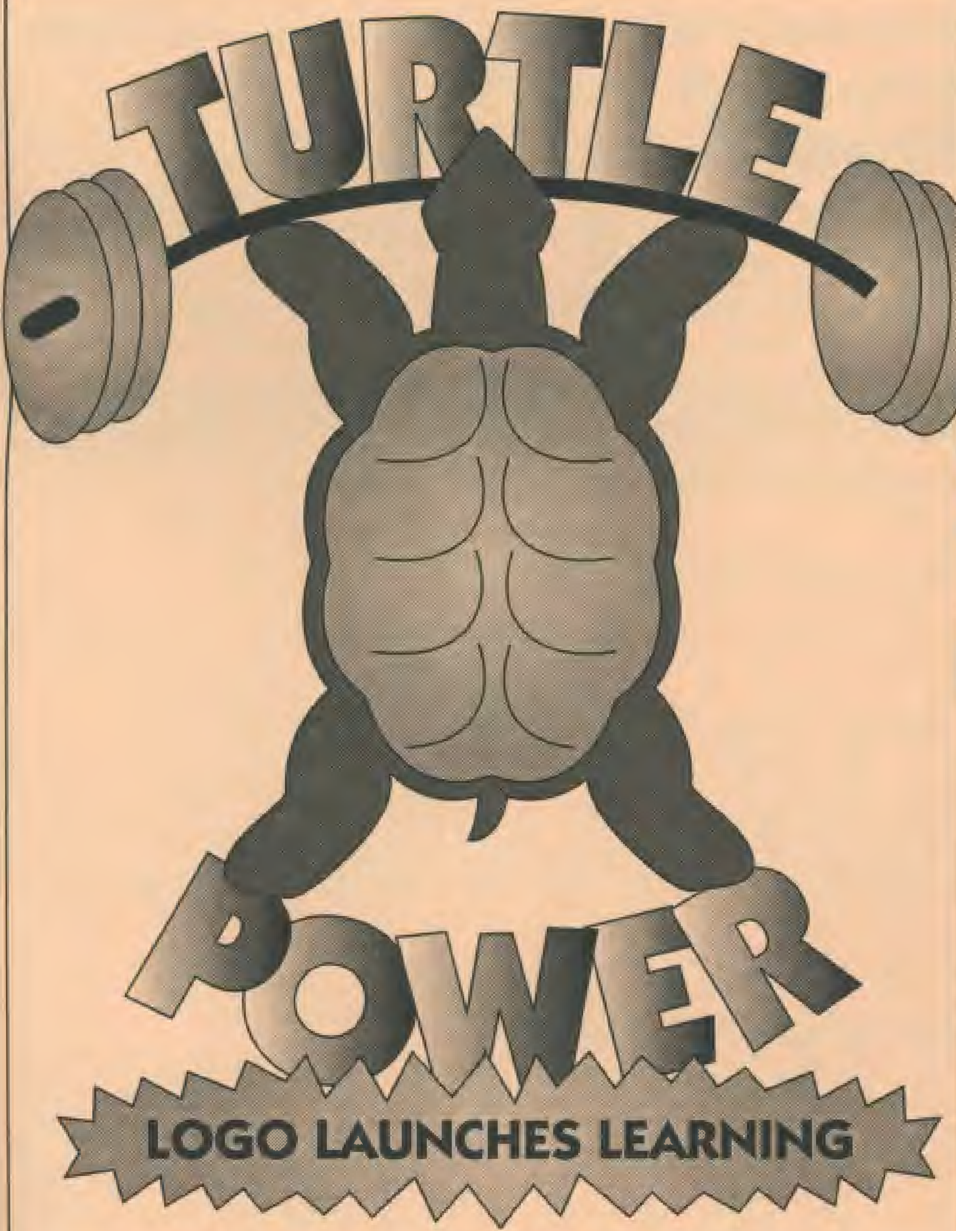
Students soon concluded that entry at a different corner would have no impact on their statistical data. Hence this part of the project was quickly discarded, although a few students made forays into this area.

Allowing exit points in addition to corners opened up interesting vistas. It was suggested that exits at the midpoints along the lengths were traditional. Those who play these table games had a particular interest in spending some of their spare time graphing examples. However, none were able to do much work beyond preliminary graphing.



Example 7

More students were struck by the possibilities offered by billiard tables with an area blocked off within the playing field. (See Example 7.) In a 10 x 8 grid, we block off a 4 x 4 area. In the first example, the symmetrical patterning is marked. In the second sample, the symmetry is expressed simply. The path of the ball in the third sample no longer possesses symmetry. These examples merely hint at possibilities. This aspect is worthy of further study.



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